Calculating pH from Solubility Product

https://gemini.google.com/app/67c151ac5055c765

Response: Here is the step-by-step solution to the problem:

1. Dissolution Equilibrium and K_{sp} Expression

The dissolution equilibrium for the sparingly soluble hydroxide $\mathrm{M(OH)_2}$ is given by:

$$\mathrm{M(OH)_2}(s) \rightleftharpoons \mathrm{M^{2+}}(aq) + 2\mathrm{OH^-}(aq)$$

The solubility product constant expression is:

$$K_{sp} = [\mathrm{M^{2+}}][\mathrm{OH^-}]^2$$

2. Calculate K_{sp} using the First Condition

At the initial state:

- $\mathrm{pH} = 8.0$
- $[\mathrm{M^{2+}}] = 1.0 \times 10^{-6}\ \mathrm{mol\ dm^{-3}}$

First, find the hydroxide ion concentration $[\mathrm{OH^-}]$:

$$\mathrm{pOH} = 14.0 - \mathrm{pH} = 14.0 - 8.0 = 6.0$$

$$[\mathrm{OH^-}] = 10^{-\mathrm{pOH}} = 1.0 \times 10^{-6}\ \mathrm{mol\ dm^{-3}}$$

Now, substitute $[\mathrm{M^{2+}}]$ and $[\mathrm{OH^-}]$ into the K_{sp} expression:

$$K_{sp} = (1.0 \times 10^{-6}) \times (1.0 \times 10^{-6})^2 = 1.0 \times 10^{-18}\ \mathrm{mol^3\ dm^{-9}}$$

3. Calculate the New pH for the Second Condition

At the new state (same temperature, so K_{sp} remains constant):

- New $[\mathrm{M^{2+}}] = 1.0 \times 10^{-4}\ \mathrm{mol\ dm^{-3}}$

Using $K_{sp} = [\mathrm{M^{2+}}][\mathrm{OH^-}]^2$:

$$1.0 \times 10^{-18} = (1.0 \times 10^{-4}) \times [\mathrm{OH^-}]^2$$

$$[\mathrm{OH^-}]^2 = \frac{1.0 \times 10^{-18}}{1.0 \times 10^{-4}} = 1.0 \times 10^{-14}$$

$$[\mathrm{OH^-}] = \quad 1.0 \times 10^{-14} = 1.0 \times 10^{-7}\ \mathrm{mol\ dm^{-3}}$$

Next, determine pOH and pH:

$$\mathrm{pOH} = -\log_{10}(1.0 \times 10^{-7}) = 7.0$$

$$\mathrm{pH} = 14.0 - \mathrm{pOH} = 14.0 - 7.0 = 7.0$$

Correct Option

Correct Option: (4) 7.0
